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Soft scalar masses in supergravity with horizontal $U(1)_X$ gauge symmetry ¹

E. Dudas

CEA, Service de Physique Théorique, CE-Saclay
F-91191 Gif-sur-Yvette Cedex, FRANCE

S. Pokorski ²

Max-Planck Institut für Physik Werner-Heisenberg-Institut
Föhringer Ring 6, D-80805 München, GERMANY

C.A. Savoy

CEA, Service de Physique Théorique, CE-Saclay
F-91191 Gif-sur-Yvette Cedex, FRANCE

Abstract

In supergravity with modular invariance and horizontal $U(1)_X$ gauge symmetry there is a relation between modular weights and $U(1)_X$ charges. The soft scalar masses are then strongly correlated with Yukawa matrices. The implications for FCNC are discussed.

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1. The observed suppression of flavour changing neutral current (FCNC) transitions is nicely explained in the Standard Model (SM) by GIM mechanism. At the same time it is a very strong constraint on physics beyond the SM which, in general, may provide new mechanisms for FCNC transitions. The supersymmetric extensions of the SM do indeed contain additional contributions to FCNC transitions from sfermion exchange in loop diagrams. Such effects are generically suppressed only as $O\left(\frac{M_Z}{M_{\tilde{f}}}\right)$ where $M_{\tilde{f}}$ is a typical sfermion mass matrix entry. They can be potentially dangerous for FCNC transitions if, in the basis in which fermion mass matrices are diagonal, the sfermion mass matrices have large flavour off-diagonal entries. The problem is aggravated by the absence of any reliable theory for calculating the soft supersymmetry breaking terms. Thus, the solution to the FCNC problem in supersymmetry remains at the level of speculations which go, essentially, in the following directions.

Most often explored is the ansatz about universal soft susy breaking terms at the GUT (or more likely -Planck M_P) scale. This indeed occurs in flavour blind supersymmetry breaking scenarios in supergravity [1] or superstrings [2], [3] (e.g. "dilaton breaking"). At the electroweak scale we obtain then the flavour dependent effects in the sfermion sector only of the order of the Cabibbo-Kobayashi-Maskawa mixing, consistently with observations. Recently, very interesting progress has been achieved in studying such "minimal" FCNC effects in the framework of GUT's [4]. If we abandon the universality ansatz, we face the problem of explaining in another way the approximate simultaneous diagonality, in the same basis, of the fermion and sfermion mass matrices. For squarks there is the possibility to wash out the flavour non-universality by large flavour blind renormalization effects from the scale where supersymmetry is broken to a low-energy scale [5], [6]. Such effects are, however, much weaker in the slepton sector and cannot explain the smallness of FCNC unless flavour dependence of the soft terms at the large scale is for some reason strictly controlled. It is then conceivable that the pattern of both types of mass matrices is simultaneously determined by some symmetries of the lagrangian [7], originally introduced in order to understand the hierarchy of the fermion masses and mixings [8]. (Another recent suggestion is the dynamical allignement [9] of the fermion and sfermion mass matrices at the electroweak scale.)

Recently [10]-[14], there is a revival of interest in explaining the pattern of fermion masses and mixing by postulating a horizontal $U(1)_X$ gauge symmetry, spontaneously broken at a large scale M . The $U(1)_X$ charges are assigned to fermions in such a way that only a small number of Yukawa interactions is allowed by the symmetry. The remaining effective Yukawa vertices are generated through non-renormalizable couplings of the fields which are SM singlets but carry horizontal charge and spontaneously break $U(1)_X$. These couplings are suppressed by powers of a small parameter, ε^{n_i} , where the powers n_i depend on the $U(1)_X$ charge assignment. In the context of supersymmetric models with “stringy” $U(1)_X$ symmetry [15]-[16], this mechanism of fermion mass generation shows an interesting connection between phenomenologically viable mass pattern and the Green-Schwarz mechanism of anomaly cancellation, which successfully predicts the Weinberg angle [17].

In this letter we study the predictions for the soft supersymmetry breaking terms which follow from the horizontal $U(1)_X$ symmetry approach to fermion masses and their implications for FCNC transitions. We work with the effective supergravity Lagrangian generic for orbifold models of string compactification, that is we impose on it the spectrum and the symmetries of the latter. This implies an interesting relation between $U(1)$ charges and the modular weights of the matter fields, with implications for the soft supersymmetry breaking terms and FCNC effects.

2. The relevant low energy limit of the superstring models are described by the $N = 1$ supergravity defined by the Kähler function K , the superpotential W and the gauge kinetic function f . The generic fields present in the zero-mass string spectrum contain an universal dilaton S , moduli fields generically denoted by T_α and matter chiral fields ϕ^i containing the standard model particles. A crucial role in the following discussion will be played by the target-space modular symmetries $SL(2, Z)$ [18] associated with the moduli fields T_α ($\alpha = 1..m$), acting as $T^\alpha \rightarrow (a_\alpha T^\alpha - ib_\alpha)/(ic_\alpha T^\alpha + d_\alpha)$, with $(a_\alpha d_\alpha - b_\alpha c_\alpha) = 1$ and $a_\alpha \dots d_\alpha \in Z$. In effective string theories of the orbifold type, the matter fields Φ^i transform under $SL(2, Z)$ as $\Phi^i \rightarrow (ic_\alpha T^\alpha + d_\alpha)^{n_i^{(\alpha)}} \Phi^i$, where the $n_i^{(\alpha)}$ are called the modular weights of the fields ϕ^i with respect to the modulus T^α . We define the *overall modular weight* of the field Φ^i by $n_i = \sum_\alpha n_i^{(\alpha)}$. These modular transformations, which are symmetries of the supergravity theory, can be viewed as a particular type of Kähler transformations.

We consider the MSSM model, which we take to be the minimal model obtained in the low-energy limit of the superstring models, plus the horizontal gauge group with one singlet ϕ of charge X_ϕ . We denote the matter fields by capitals Φ^i , the corresponding $U(1)_X$ charges by small letters φ_i and we define $\varphi'_i = -\frac{\varphi_i}{X_\phi}$. The $U(1)_X$ invariant superpotential W and the Kähler potential K read,

$$\begin{aligned}
W &= \sum_{ij} \left[Y_{ij}^U \theta(q'_i + u'_j + h'_2) \left(\frac{\phi}{M} \right)^{q'_i + u'_j + h'_2} Q^i U^j H_2 + \right. \\
&\quad Y_{ij}^D \theta(q'_i + d'_j + h'_1) \left(\frac{\phi}{M} \right)^{q'_i + d'_j + h'_1} Q^i D^j H_1 + \\
&\quad \left. Y_{ij}^E \theta(\ell'_i + e'_j + h'_1) \left(\frac{\phi}{M} \right)^{\ell'_i + e'_j + h'_1} L^i E^j H_1 \right], \\
K &= K_0(T^\alpha, \bar{T}^{\bar{\alpha}}) - \ln(S + \bar{S}) + \prod_{\alpha=1}^p t_\alpha^{n_\alpha^{(\alpha)}} \bar{\phi} \phi \\
&\quad + \sum_{\Phi^i = Q^i, U^i, D^i, L^i, E^i} K_{ij} \bar{\Phi}^i \Phi^j, \\
K_{ij} &= \delta_{ij} \prod_{\alpha=1}^p t_\alpha^{n_j^{(\alpha)}} + Z_{ij} \left[\theta(\varphi'_i - \varphi'_j) \prod_{\alpha=1}^p t_\alpha^{n_j^{(\alpha)}} \left(\frac{\bar{\phi}}{M} \right)^{\varphi'_i - \varphi'_j} + \right. \\
&\quad \left. \theta(\varphi'_j - \varphi'_i) \prod_{\alpha=1}^p t_\alpha^{n_i^{(\alpha)}} \left(\frac{\phi}{M} \right)^{\varphi'_j - \varphi'_i} \right] + \dots
\end{aligned} \tag{1}$$

In (1), M is a large mass scale of the order M_P , t_α are the real part of the p moduli fields T_α and the dots stand for higher order terms in the fields ϕ and $\bar{\phi}$. Note the flavour non-diagonal terms in the Kähler potential, proportional to the numbers Z_{ij} ($Z_{ii} = 1$ by a choice of normalization). The coefficients Z_{ij} , Y_{ij}^U , Y_{ij}^D , Y_{ij}^E , allowed by the symmetries are supposed to be naturally of $O(1)$.

In order to impose the modular symmetries, let us first define $n_0^{(\alpha)}$ by the modular transformations of the Kähler potential for the modular fields, $K_0 \rightarrow K_0 + n_0^{(\alpha)} \ln |ic_\alpha T_\alpha + d_\alpha|^2$, which is a Kähler transformation. A typical

example, with $n_0^{(\alpha)} = (3/p)$, is

$$K_0 = -\frac{3}{p} \sum_{\alpha=1}^p \ln t_\alpha. \quad (2)$$

Allowing for possible flavour-blind automorphic functions of weight $n_W^{(\alpha)}$ in the Yukawa couplings, the modular invariance of $G = K + \ln|W|^2$ gives, written explicitly for the quarks and leptons

$$\begin{aligned} \theta(q'_i + u'_j + h'_2) \left[(q'_i + u'_j + h'_2) n_\phi^{(\alpha)} + n_{q_i}^{(\alpha)} + n_{u_j}^{(\alpha)} + n_{h_2}^{(\alpha)} + n_0^{(\alpha)} + n_W^{(\alpha)} \right] &= 0, \\ \theta(q'_k + d'_l + h'_1) \left[(q'_k + d'_l + h'_1) n_\phi^{(\alpha)} + n_{q_k}^{(\alpha)} + n_{d_l}^{(\alpha)} + n_{h_1}^{(\alpha)} + n_0^{(\alpha)} + n_W^{(\alpha)} \right] &= 0, \\ \theta(l'_m + e'_n + h'_1) \left[(l'_m + e'_n + h'_1) n_\phi^{(\alpha)} + n_{l_m}^{(\alpha)} + n_{e_n}^{(\alpha)} + n_{h_1}^{(\alpha)} + n_0^{(\alpha)} + n_W^{(\alpha)} \right] &= 0. \end{aligned} \quad (3)$$

Using (3), we get in a straightforward way the relation

$$(q_i - q_j) n_\phi^{(\alpha)} = X_\phi \left(n_{q_i}^{(\alpha)} - n_{q_j}^{(\alpha)} \right), \quad (4)$$

as a consequence of the existence of the Yukawa couplings Y_{ik}^U and Y_{jk}^U in the superpotential. Similar relations are obtained by replacing q_i by u_i, d_i, l_i, e_i . The relations (4) gives a surprising connection between the modular weights and the $U(1)_X$ charges. The eq.(4) are at the same time the modular invariance conditions for the existence of the flavour non-diagonal terms in the Kähler potential. More exactly, imposing in the off-diagonal parts of the Kähler potential (1) the same weight for the modular transformations of the chiral and the antichiral parts we get once again eq. (4).

If eq.(4) are not satisfied, modular invariance of the superpotential implies zeroes in the Yukawa matrices and in the off-diagonal entries of the Kähler metric. These type of zeroes must be distinguished from the ones given by $U(1)_X$ invariance and the holomorphicity of the superpotential W and described by the θ -functions in (1). We could try to construct phenomenologically interesting models in this way, in the spirit of ref.[19]. An useful rule in this respect is the following. Consider a 2×2 sub-matrix with three non-zero entries. Then non-vanishing of the fourth one is automatically consistent with modular invariance, as a straightforward consequence

of eq.(4). This is similar, even though has a very different origin, to the rule of ref.[20]. Using consistently this rule, the only configurations with zeroes which follow from modular invariance of the potential (1) (up to permutations of lines and columns which reduce to mere permutations of different

generations) are of the type $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$ and $\begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$.

The physical Yukawa couplings, \hat{Y} , are obtained by the canonical normalization of the kinetic terms, which requires the redefinition of the fields $\hat{\Phi}^i = e_j^i \Phi^j$ where the vielbein $e_j^i(t_\alpha, \phi)$ verify

$$K_{\bar{i}j} = \delta_{\bar{k}k} \bar{e}_{\bar{i}}^{\bar{k}} e_j^k = \prod_{\alpha=1}^p t_\alpha^{(n_i^{(\alpha)} + n_j^{(\alpha)})/2} \left(\delta_{\bar{i}j} + Z_{\bar{i}j} \hat{\varepsilon}^{|\varphi'_i - \varphi'_j|} \right) \quad (5)$$

where (3) and (4) are assumed and the small parameter $\hat{\varepsilon} = \prod_{\alpha} t_\alpha^{n_\phi^{(\alpha)}/2} \frac{\langle \phi \rangle}{M}$ serves to estimate the size of the fermion and scalar mass matrix elements (if, for some $(\bar{i}j)$, (4) is not fulfilled, the coefficient vanishes). The potential effect of these field redefinition on (5) is to remove the eventual zeroes in the Yukawa matrices (Examples of this type of a phenomenological interest can be found in [13]). However, due to the fact that modular symmetry zeroes in Yukawa matrices imply zeroes in the corresponding off-diagonal elements of the Kähler metrics, the zero textures of the above matrices are preserved after multiplication by the vielbein. Phenomenologically, they can accommodate the fermion masses and one mixing angle, but they cannot explain the whole V_{CKM} matrix. Hence, for the quarks, the relations (4) must be imposed for *all* the indices (i, j) (of course, zeroes due to the holomorphicity of W can be filled as in ref.[13]).

The eqs.(4) generate, as explained in the next sections, many equations relating the soft-breaking terms in the low-energy theory. Interestingly enough, it provides a relation between the hierarchies within Yukawa coupling matrix elements and modular weights. Indeed, with canonical normalization for the fermion fields, one gets, if $n_\phi^{(\alpha)} \neq 0$, e.g.,

$$\hat{Y}_{ij}^U \sim \hat{\varepsilon}^{(n_{q_i}^{(\alpha)} - n_{q_3}^{(\alpha)} + n_{u_j}^{(\alpha)} - n_{u_3}^{(\alpha)})/n_\phi^{(\alpha)}} \quad (6)$$

as well as analogous relations for the matrix elements of Y^D and Y^E (with an additional factor Y_{33}^D on the r.h.s.).

It would be interesting to compare this approach with that developed in [21], where the role of the horizontal symmetry is played by the modular symmetries.

3. The spontaneous breaking of local supersymmetry gives rise to a low-energy global supersymmetric theory together with terms that explicitly break supersymmetry, but in a soft way. The signal of supersymmetry breaking is provided by non zero vev's of the auxiliary components of the chiral superfields $F^a = e^{\frac{G}{2}} G^a$, where $G^a = K^{ab} \partial_b G$. We consider only the case of zero tree level cosmological constant, i.e., we impose $\langle G^a G_a \rangle = 3$ and the order parameter for the supergravity breaking is provided by the gravitino mass $m_{3/2}^2 = e^G$. A complete scenario of supersymmetry breaking is still missing. A pragmatic attitude was taken in [5], where a parametrization of the supersymmetry breaking was proposed, quite independent of its specific mechanism. The fields which participate at the supergravity breaking were assumed to be the moduli T^α and the dilaton S . The parametrization is

$$\begin{aligned} G^\beta &= \sqrt{3} \Theta_\beta t_\beta, \\ G^\beta G_\beta &= 3 \cos^2 \theta, \\ G^S G_S &= 3 \sin^2 \theta. \end{aligned} \tag{7}$$

The angle θ and the Θ_α parametrize the direction of the goldstino in the T_α, S space. The normalization of the Θ_α is fixed by (7). In the presence of the $U(1)_X$ symmetry spontaneously broken close to the Planck scale there is an additional contribution to supersymmetry breaking with $\langle G_\phi G^\phi \rangle = \hat{\varepsilon}^2 M^2 / M_P^2$.

The soft terms are computed from the usual expressions of supergravity, but with the flavour non-diagonal Kähler potential, eq.(1). It is worth noticing that only the lowest power of $\hat{\varepsilon}$ or ϕ have been defined in (1) and the calculations are to remain consistent with this approximation. It goes without saying that the predictions herebelow have been derived to the lowest power of $\hat{\varepsilon}$. Since the soft parameters are relevant for low-energy phenomenology, it is more appropriate to express them *after the field redefinition* that brings the kinetic terms to their canonical forms as consistently done herebelow.

Let us first consider the soft scalar masses that, up to a term proportional to the square of the fermion mass matrices, have the standard supergravity

expression

$$\widetilde{m}_{ij}^2 = \left(\delta_{ij} - G^\alpha \hat{R}_{\bar{i}j\bar{\beta}\alpha} G^{\bar{\beta}} \right) m_{3/2}^2 + g \varphi_i \delta_{ij} < D >, \quad (8)$$

where $\hat{R}_{\bar{i}j\bar{\beta}\alpha}$ is the Riemann tensor of the Kähler space (with fermion indices for canonical fields) and $< D >$ stands for the contribution from the D-term of the $U(1)_X$ gauge group, $D = g \left(X_\phi \left| \hat{\phi} \right|^2 + \varphi_i \left| \hat{\Phi}_i \right|^2 + \xi \right)$ where $\xi = \left(\frac{M_p^2}{192\pi^2} \right) Tr X$ [15]. In the simple case with only one singlet field ϕ , the minimization of the potential gives

$$d = g X_\phi < D > = -\widetilde{m}_\phi^2 = - \left(1 + 3n_\phi^{(\alpha)} \Theta_\alpha^2 \right) m_{3/2}^2. \quad (9)$$

In this one-singlet model, the $U(1)_X$ symmetry is broken by the Fayet-Il-iopoulos term ξ , and this, together with the (assumed) supersymmetry breaking along the S and T^α components, induce a component along the ϕ direction, with $< G_\phi G^\phi > = \hat{\varepsilon}^2$.

From (5), (7) and (8), one obtains the expression for the soft scalar mass matrices as follows

$$\frac{1}{m_{3/2}^2} \widetilde{m}_{ij}^2 = \left(1 + 3n_i^{(\alpha)} \Theta_\alpha^2 - \varphi'_i d \right) \delta_{ij} + 3 \left| \varphi'_i - \varphi'_j \right| Z_{ij} n_\phi^{(\alpha)} \Theta_\alpha^2 \hat{\varepsilon}^{|\varphi'_i - \varphi'_j|}. \quad (10)$$

Remarkably enough, the contribution from supersymmetry breaking along the ϕ direction to (10) vanishes.

On account of the non-diagonal form of the Kähler potential, the scalar mass matrices are not diagonal as in the usual computations in the literature. There are two important aspects of this result. As was discussed in [7], the horizontal $U(1)_X$ symmetry has the virtue of suppressing the flavour off-diagonal terms in the sfermion mass matrices in a way correlated with the Yukawa matrices. Moreover, in our model which combines the $U(1)_X$ symmetry with modular invariance the coefficients of all the entries, including the diagonal ones, are very constrained and $U(1)_X$ charge dependent.

The soft terms (10) depend on the parameters $n_i^{(\alpha)}$, θ , Θ_α , d , etc. In spite of that, one obtains from (4) a striking result for the squark and slepton mass differences. Indeed, by inserting (9) and (4) into (10) one obtains for $\Phi^i = Q^i, U^i, D^i, L^i, E^i$, the predictions:

$$\widetilde{m}_{ii}^2 - \widetilde{m}_{jj}^2 = (\varphi'_i - \varphi'_j) m_{3/2}^2. \quad (11)$$

The simple form of this result is due to a conspiracy between moduli field and D-term contributions in this one-singlet model.

Also, combining (3), (4) and (10) and introducing the tree-level gaugino masses $M = \sqrt{3} \sin \theta m_{3/2}$, we obtain the relations (assuming $n_W^{(\alpha)} = 0$)

$$\widetilde{m}_{q_i}^2 + \widetilde{m}_{u_j}^2 + \widetilde{m}_{h_2}^2 = M^2 + (q'_i + u'_j + h'_2) m_{3/2}^2 . \quad (12)$$

Similar relations are obtained for d-type squarks and sleptons.

There is another relation for soft masses which follows from the phenomenological approximate equation, $\det \hat{Y}^D = \det \hat{Y}^L$. The latter translates into a relation for the $U(1)_X$ charges, $\sum_i (q_i + d_i) = \sum_i (\ell_i + e_i)$. By subtracting the second and the third equations in (3) one gets the relation $\sum_i (n_{q_i}^{(\alpha)} + n_{d_i}^{(\alpha)}) = \sum_i (n_{\ell_i}^{(\alpha)} + n_{e_i}^{(\alpha)})$. Hence, from (10) one obtains the mass sum rule,

$$\sum_i (\widetilde{m}_{\ell_i}^2 + \widetilde{m}_{e_i}^2) = \sum_i (\widetilde{m}_{q_i}^2 + \widetilde{m}_{d_i}^2) . \quad (13)$$

All these equations for scalar masses are to be understood at energies of the order M_P , and lead to low energy relations after renormalization.

The non-diagonal terms in K and W affect the trilinear soft terms V_{ijk} , too. The general expression for the trilinear terms corresponding to fields with $\langle G^i \rangle = \langle G_i \rangle = 0$ is [22],

$$V_{ijk} = \left[(G^\alpha D_\alpha + 3) \frac{W_{ijk}}{W} \right] m_{3/2}^2 , \quad (14)$$

where D stands for the covariant derivative in the Kähler manifold. Once again we work with canonical normalization of the scalar fields. With this convention and in the leading order of the small parameter $\hat{\varepsilon}$ the connections in the covariant derivatives in (14) take the simple form,

$$G^\alpha \Gamma_{\alpha i}^j = n_i^{(\alpha)} \Theta_\alpha \delta_i^j + \frac{1}{2} |\varphi'_i - \varphi'_j| n_\phi^{(\alpha)} \Theta_\alpha \hat{\varepsilon}^{|\varphi'_i - \varphi'_j|} Z_{i\bar{j}} . \quad (15)$$

The final result for the triscalar coefficient V_{ia}^U , for example, reads

$$\begin{aligned}
\frac{1}{m_{3/2}} \hat{V}_{ia}^U &= \sqrt{3} \left[-\sin \theta + (q'_i + u'_a + h'_2) n_\phi^{(\alpha)} \Theta_\alpha \right] \hat{Y}_{ia}^U \\
&\quad - \sqrt{3} \left(t_\alpha \frac{\partial \ln \hat{Y}_{ia}^U}{\partial T_\alpha} + t_\alpha \partial_\alpha K_0 - n_W^{(\alpha)} - n_0^{(\alpha)} \right) \Theta_\alpha \hat{Y}_{ia}^U + \\
&\quad \frac{\sqrt{3}}{2} n_\phi^{(\alpha)} \Theta_\alpha \left(\sum_j |q'_i - q'_j| Z_{i\bar{j}} \hat{Y}_{ja} \hat{\varepsilon}^{|q'_i - q'_j|} + \sum_b |u'_b - u'_a| Z_{a\bar{b}} \hat{Y}_{ib} \hat{\varepsilon}^{|u'_b - u'_a|} \right) \\
&\quad - (q'_i + u'_a + h'_2) \hat{Y}_{ia}^U - (q'_i - q'_j) \theta(q'_i - q'_j) Z_{i\bar{j}} \hat{Y}_{ja} \hat{\varepsilon}^{|q'_i - q'_j|} \\
&\quad - (u'_a - u'_b) \theta(u'_a - u'_b) Z_{a\bar{b}} \hat{Y}_{ib} \hat{\varepsilon}^{|u'_a - u'_b|}
\end{aligned} \tag{16}$$

and similar expressions hold for V^D and V^L with obvious replacements. Notice that the matrices \hat{Y} have hierarchical entries as expressed by (6). The last terms in (16) come from $G^\phi D_\phi$, namely, from supersymmetry breaking along the ϕ direction.

4. We now turn to the computation of the FCNC effects in our model. The fermion masses and the Cabibbo-Kobayashi-Maskawa matrix are obtained by diagonalization of the mass matrices $U_L \hat{m}^U \hat{U}_R^+ = \text{diag}(m_u, m_c, m_t)$ and the rotation matrices D_L, D_R, L_L, L_R , are defined analogously. The sfermion masses are given by 6×6 matrices $(\widetilde{M}^F)^2$, $F = U, D, L$, which can be divided into 3×3 sub-matrices

$$(\widetilde{M}^F)^2 = \begin{pmatrix} (\widetilde{M}^F)_{LL}^2 & (\widetilde{M}^F)_{LR}^2 \\ (\widetilde{M}^F)_{LR}^{2+} & (\widetilde{M}^F)_{RR}^2 \end{pmatrix}. \tag{17}$$

The relevant quantities for the FCNC processes are sfermion off-diagonal mass matrix elements in the basis in which fermion masses are diagonal, i.e.

$$(\delta_{MN}^F)_{ij} = \frac{1}{\widetilde{m}^2} \left[F_M (\widetilde{M}^F)_{MN}^2 F_N^+ \right]_{ij}, \tag{18}$$

where $M, N = L, R$, F denotes the rotation matrices for fermion F and \widetilde{m}^2 is an averaged sfermion mass squared. There exists experimental bounds for the quantities δ [6]. Typically, $(\delta_{MM}^{d,u})_{ij} \leq \mathcal{O} \left(\infty'^{-\infty} \left(\frac{\infty}{\infty T |v|} \right) \right)$ with,

however, much weaker constraint for the (2,3) element in the down sector and no constraints for the (1,3) and (2,3) elements in the up sector. The bounds on the combinations

$$\delta_{ij}^F = \sqrt{(\delta_{LL}^F)_{ij} (\delta_{RR}^F)_{ij}} \quad (19)$$

are one order of magnitude stronger (on the analogous elements) with the strongest bound $\delta_{12}^d \leq 8 \times 10^{-3} (\tilde{m}/1TeV)$. The best bounds in the lepton sector are $(\delta_{MM}^\ell)_{12} \leq 10^{-1} \left(\frac{\tilde{m}_\ell}{1TeV}\right)^2$. We do not discuss the bounds on δ_{LR} . They constrain the trilinear V soft terms. As was remarked in [7], the $U(1)_X$ symmetry gives typically $(\delta_{LR}^q)_{ij} \sim \frac{1}{\tilde{m}} \sqrt{m_i^q m_j^q}$ leading to very small contributions to FCNC.

These bounds apply at the electroweak scale. They have to be satisfied by the soft scalar masses after their renormalization group running from the large scale at which they are determined by a deeper theory. Eq.(10) gives the sfermion mass matrices at the large (GUT, string) scale, in the basis defined by the $U(1)_X$ symmetry. The entries in (10) are determined by the $U(1)_X$ charge assignment of sfermions. Here is the link with the quark Yukawa matrices, determined by the same charge assignment of the $U(1)_X$ symmetry to fermions.

To go further in the discussion of FCNC one has to specify the model for Yukawa matrices. Let us consider models with one singlet field. The acceptable quark Yukawa matrices and the $U(1)_X$ charge assignments are listed in ref. [13] (they include the original proposal of Froggatt-Nielsen). To illustrate the flavour changing problem in these models, we explicitly calculate δ_{12} 's, for which the constraints are strongest. Since all those solutions satisfy the relations $(q'_1 - q'_2) \geq 0, (d'_1 - d'_2) \geq 0, (u'_1 - u'_2) \geq 0$, one derives the following results:

$$\begin{aligned} (\delta_{LL}^{u,d})_{12} &= (q'_1 - q'_2)(F_{12}^q) \epsilon^{q'_1 - q'_2} , \\ (\delta_{RR}^u)_{12} &= (u'_1 - u'_2)(F_{12}^u) \epsilon^{u'_1 - u'_2} , \\ (\delta_{RR}^d)_{12} &= (d'_1 - d'_2)(F_{12}^d) \epsilon^{d'_1 - d'_2} , \end{aligned} \quad (20)$$

where the F' s are obtained from their values at the large scale M ,

$$F_{ij}^u(M_X) = \frac{O(m_{3/2}^2)}{(\tilde{m}_i^2 + \tilde{m}_j^2)} . \quad (21)$$

through the renormalization group running down to the electroweak scale. Notice that $\epsilon^{q'_1 - q'_2 + u'_1 - u'_2} \sim \frac{m_u}{m_c}$ and $\epsilon^{q'_1 - q'_2 + d'_1 - d'_2} \sim \frac{m_d}{m_s}$ and, consequently, $(\delta_{12}^u)^2$ and $(\delta_{12}^d)^2$ can be reexpress directly in terms of the quark masses. For example,

$$(\delta_{12}^d)^2 = (q'_1 - q'_2)(d'_1 - d'_2)(F_{12}^q)(F_{12}^d)\frac{m_d}{m_s}. \quad (22)$$

The final leading order result is a sum of two terms, with the same powers of the small parameter ϵ and proportional to the charge differences. One is given by the original off-diagonal terms in (10) and the other is proportional to the splittings among the diagonal entries multiplied by the rotation angles, which are explicitly given in [13]. The vanishing of the leading term in the limit of universal diagonal terms in the sfermion mass matrices reflects an improved allignement of fermion and sfermion masses due to the $U(1)_X$ symmetry.

In this class of models, the required suppression of FCNC can be achieved if the functions F are small enough or if *some of the charge differences in eq.(22) vanish*. For instance, $(\delta_{12}^d)^2 \leq 10^{-4} \left(\frac{\tilde{m}}{1\text{TeV}}\right)^2$ requires $(F_{12}^q)(F_{12}^d) \leq 1.6 \cdot 10^{-3}$ for $\tilde{m} \sim 1 \text{ TeV}$ and $(F_{12}^q)(F_{12}^d) \leq 0.6 \cdot 10^{-4}$ for $\tilde{m} \sim 0.2 \text{ TeV}$ or one of the equations $(q'_1 - q'_2) = 0$, $(d'_1 - d'_2) = 0$ has to be satisfied. For small values of the functions F one needs large renormalization effects in the diagonal squark masses, which requires a sizeable dilaton supersymmetry breaking .

The second option can also be realized with acceptable quark mass matrices. Following the classification of ref. [13] of all acceptable models with one singlet ϕ we see that the condition $d_1 = d_2$ is satisfied in two models defined by

$$\begin{aligned} (q'_1 - q'_3) = 3, (q'_2 - q'_3) = 2, (u'_1 - u'_3) = 5, (u'_2 - u'_3) = 2, (d'_1 - d'_3) = 1, \\ (q'_1 - q'_3) = 4, (q'_2 - q'_3) = 3, (u'_1 - u'_3) = 4, (u'_2 - u'_3) = 1, (d'_1 - d'_3) = 1. \end{aligned} \quad (23)$$

For these models, $\delta_{12}^d = 0$ in the leading order and the non-leading terms are within the experimental bounds if $(F_{12}^q)(F_{12}^d) \leq .008$. The limits on $(\delta_{MM}^{u,d})_{12}$ are also satisfied. So we conclude that in such models the FCNC effects are weakened, without asking for a large dilaton contribution to supersymmetry breaking.

FCNC effects in the lepton sector are potentially more dangerous as no strong interaction renormalization effects can wash out the flavour off-diagonal terms present at the large scale. However, due to our ignorance on

the leptonic mixing angles, the lepton mass models are much less constrained and many more charge assignments are possible. Assuming eq.(4) to hold for all of the off-diagonal terms in the lepton sector we get, analogously to (22)

$$\left(\delta_{12}^\ell\right)^2 = (l'_1 - l'_2)(e'_1 - e'_2) F_{12}^l F_{12}^e \left(\frac{m_e}{m_\mu}\right)^n, \quad (24)$$

where $F^{l,e}$ are similar to $F^{q,d}$ with some obvious replacements and generically $n = 1, 2, 3$. The freedom present in the lepton sector makes it relatively easy to satisfy FCNC constraints, but the analysis must be done model by model.

The relation (22) changes for models with more than one singlet and/or negative charge differences.

5. In this letter, we have discussed effective supergravity models with horizontal $U(1)_X$ gauge symmetry. It has been shown that the horizontal symmetry and the modular invariances have to be correlated: horizontal charges and modular weights must satisfy eq.(4) in order to allow for non-vanishing Yukawa couplings. The same relations can be viewed as the conditions for the existence of flavour non-diagonal terms in the Kähler potential. In turn, the soft supersymmetry breaking terms depend on the $U(1)_X$ charges and are correlated with Yukawa matrices. This results in a predictive framework for the soft masses. The FCNC problem for the squarks can be eased without asking for a large dilaton contribution to supersymmetry breaking. A systematic phenomenological discussion of different models along these lines is certainly worthwhile.

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